A Tractable Framework for Power Consumption in Femtocell networks

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***Abstract*—In this paper, we use stochastic geometry to analyze the transmit power consumption in femtocell network. The femtocell base stations (FAPs) are assumed to follow homogeneous Poisson point process. Firstly, we propose a power adjusting strategy for FAPs according to their relative distance from the macro base station and then quantitatively analyze its mean total power consumed in the femtocell network. There're possibilities that the signal strength from the MBS is obstructed such that the FAP can't achieve its distance. In order to combat this issue, neighboring FAPs are used to determine this power-undefined FAP. If there are no users in a specific FAP's coverage area, this FAP should turn down or off their transmit power and the probability that the FAP's "silent" probability is also achieved. All results are tractable mathematically. This paper presents a tractable framework for the power consumption in the femtocell network.**

***Keywords*-femtocell; stochastic geometry; power consumption; Poisson point process;**

I. INTRODUCTION

Femtocell access points (FAPs), also called "home base station", are crucial for next generation cellular networks. Femtocells can effectively improve the capacity of a cellular network without any significant increase in the network management costs [1-2]. FAPs are usually installed by users and deployed in indoor environments. These low-power base stations provide a limited coverage area and connect to the operators' core networks through digital subscriber line (DSL), cable broadband connection, or even wireless links [3].

The interest in femtocells in the mobile operator community continues to grow, and the commercial deployments have increased to 41 in 23 countries during 2012 [4]. It is estimated by Informa Telecoms & Media that the deployments of small cell market would reach 91.9 million by 2016, with femtocells accounting for more than 80%. It is expected the FAPs would be widely deployed in the near future. Though femtocells can offload traffic from the macro stations and are crucial for next generation mobile communications, special challenges exist due to the randomness of the locations of femtocells [5].

Because femtocell base stations are installed by non-expert users, it would present very different characteristics such as the locations of FAPs. The randomness of the locations of femtocells brings us a lot of difficulties to analyze and compare. In order to quantitatively analyze the performance on femtocells, stochastic geometry has been used in some papers. In [6], the authors regard stochastic geometry as one of the four effective tools to solve small cell problems (other three are large random matrix theory, Game theory, and interference alignment and VFDM). The authors in [7] propose a tractable approach to solve the problem of SINR distribution for femtocell networks using stochastic geometry while in [8], fractional frequency reuse for OFDMA cellular networks is solved. Those works are based on homogeneous Poisson point process to model the locations of FAPs. One advantage of this approach is the ability to capture the non-uniform layout of modern cellular deployments due to topographic, demographic, or economic reasons [9]. Additionally, tractable expressions can be drawn from the Poisson model, leading to more general performance characterizations and intuition [10].

In this paper, we use stochastic geometry to analyze the power consumption in femtocell network. The femtocell base stations are assumed to follow homogeneous Poisson point process. Firstly, we propose a power adjusting strategy for FAPs according to their relative distance from the macro base station (MBS) and then quantitatively analyses its mean total power consumed in the femtocell network. There're possibilities that the signal strength from the MBS is obstructed such that the FAP can't achieve its distance. In order to combat this issue, neighboring FAPs are used to determine this power-undefined FAP. Thus the femtocell isolated probability should be pinned down first, which can be resolved by Boolean model (which is one branch of Stochastic Geometry).

II. FEMTOCELL NETWORK EMPLOYMENT

Figure 1 shows the schematic diagram about the femtocell network. In this figure, only a few number of femtocells are drawn and its coverage radius is expanded. In fact, there're a lot of FAPs in the cellular network and the coverage of each femtocell is small that has radius from about $10m$ to $20m$. In this figure, a FAP forbidden area (with radius of R_{forbid}) is defined because users are impossible to buy FAPs to enhance their experience when their signal strength from the MBS is already very high.

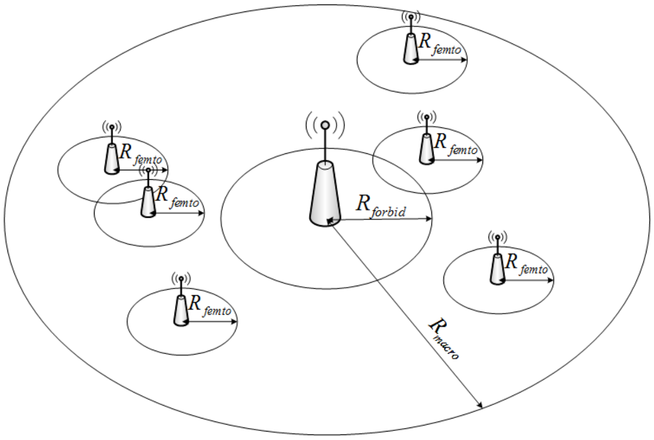

Figure 1. The system schematic diagram considered in the paper. R_{forbid} is the radius that no femtocell base station is in it, which is realistic because there's no necessity to install femto base stations there given that the macro's signal strength is strong. The locations of fetmocell base stations follow homogeneous Poisson point process from the radius R_{forbid} to R_{macro}.

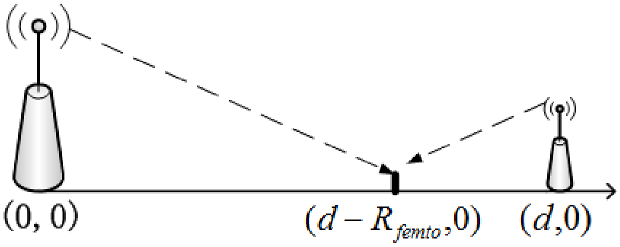

Figure 2. The ratio of MBS's signal strength to FAP's is fixed in the border.

A. Signal Propagation Model

Letting each femtocell coverage radius is equal to R_{femto} and the received signal-to-interference (SIR) is equated with some given value in the fmetocell border. Thus, the power of each femtocell base station (FAP) is totally decided by its distance from the macro base station (MBS). Suppose that the macro base station is located in (0,0) and a femto base station in (d,0) (see Figure 2). Letting $P_r^{(m)}$ denotes signal strength received from the MBS and is equated to $P_m \cdot L_m \cdot (\frac{d-R_{femto}}{d_m})^{-\alpha}$, where the d_m is the reference distance in the macro area, L_m is the path loss at the reference distance, P_m is the transmit power of the MBS and α is the outdoor path loss exponent (usually $\alpha \geq 3$). Meanwhile, the signal strength from the FAP (which is denoted as $P_r^{(f)}$)is equated to $\frac{P_f \cdot L_w \cdot L_f}{T} \cdot (\frac{R_{femto}}{d_f})^{-\beta}$, where P_f is the transmit power of the FAP which is to be resolved, T is the ratio of MBS's signal strength to FAP's when measured in the border, L_w is the wall penetration loss, d_f is reference distance in the indoor area, L_f is the corresponding path loss at the reference distance and β denotes the indoor path loss exponent. Let $P_r^{(f)} = P_r^{(m)}$ comes

$$P_f(d) = \frac{P_m \cdot L_m \cdot T}{L_w \cdot L_f} \cdot (\frac{d-R_{femto}}{d_m})^{-\alpha} / (\frac{R_{femto}}{d_f})^{-\beta}. \tag{1}$$

Given the distance from the MBS, FAP can achieve its transmit power according to (1).

III. POWER CONSUMPTION AND SCHEDULING IN FEMTOCELL NETWORK

Because we assume that the locations of FAPs follow homogeneous Poisson point process, which is realistic in the next generation telecommunication network [6-7]. The femtocell network's mean total power P_{femto} consumed is equal to $E(\sum_i P_f(|x_i|))$, where $|\cdot|$ denotes the distance from the FAP to the MBS and x_i denotes the i^{th} FAP's location. Under the condition that it follows homogeneous Poisson point process with intensity λ_{femto}, the P_{femto} can be achieved as follows:

$$\begin{aligned}
&P_{femto} = E(\textstyle\sum_i P_f(|x_i|)) = E(\int_{\mathbb{R}^d} P_f(x)\phi(dx)) \\
&= \textstyle\int_{\mathbb{R}^d} P_f(x)\Lambda(dx) = 2\pi \cdot \lambda_{femto} \cdot \int_{R_{forbid}}^{R_{macro}} P_f(r) \cdot r \cdot dr \\
&\overset{\text{simplified}}{=\!=\!=\!=} 2\pi \cdot T \cdot \lambda_{femto} \cdot \tfrac{P_m \cdot L_m}{L_w \cdot L_f} \cdot d_m^{\alpha} \cdot (\tfrac{R_{femto}}{d_f})^{\beta} \cdot \\
&\tfrac{1}{(\alpha-1)\cdot(\alpha-2)} \cdot \{(R_{macro} - R_{femto})^{-\alpha+1} \cdot (R_{femto} - \\
&(\alpha-1) \cdot R_{macro}) + (R_{forbid} - R_{femto})^{-\alpha+1} \\
&\cdot((\alpha-1) \cdot R_{forbid} - R_{femto})\}.
\end{aligned} \tag{2}$$

Clearly, equation (2) can be used to evaluate the power consumed by femtocell base stations according to the preceding power adjusting strategy.

If each transmit power is fixed, the transmit power must be set high enough to satisfy the most terrible situation, that is, in the situation of installed near the MBS. In order to guarantee the successful business employment of femtocell base stations, the advertised coverage must be achieved, thus in this case, the mean total power consumed by femtocell network is

$$P_{femto}^1 = \pi \cdot (R_{macro}^2 - R_{femto}^2) \cdot \lambda_{femto} \cdot P_f(R_{forbid}). \tag{3}$$

And the power difference between the power adjusting strategy and the fixed power strategy is:

$$\Delta_{femto} = P_{femto}^1 - P_{femto}. \tag{4}$$

When there's no user in the particular femtocell area, this femtocell base station should turn down or even off its transmit power. The probability of a FAP that is off is:

$$\begin{aligned}
P_{off} &= P\{\Phi(A) = 0\} = e^{-\Lambda(A)} \\
&= e^{-\lambda_{user} \cdot \pi \cdot (R_{macro}^2 - (R_{forbid} - R_{femto})^2)}.
\end{aligned} \tag{5}$$

And $P_{on} = 1 - P_{off}$. Because the FAPs can only be installed outside the radius of R_{forbid} and thus the users can only be located outside the radius of $R_{forbid} - R_{femto}$ and preceding equation is got. The mean total power consumed of femtocell network is then:

$$P^2_{femto} = \int_{\mathbb{R}^d} P_{on} \cdot P_f(x)\Lambda(dx) = P_{on} \cdot P_{femto}. \tag{6}$$

It has chance that FAPs be obstructed such that they can't received the MBS's signal to estimate its distance from the MBS. Thus the power adjusting strategy cannot work. Let P_{obst} as the probability that a FAP can't determine its distance. If there's no user in the FAP's area, this FAP's transmit power is assumed to 0, thus it's defined. In here, the power-undefined FAP is referred to a FAP that has users in service in its area and can't determine its distance meanwhile. Thus, the P_{undef} that is the probability of power-undefined FAPs can be induced as:

$$\begin{aligned} P_{undef} &= P\{\text{FAP is undefined} \mid \text{no serving user}\} \\ &\quad \cdot P\{\text{no serving user}\} + P\{\text{FAP is undefined} \mid \\ &\quad \text{has serving user}\} \cdot P\{\text{has serving user}\} \\ &= P_{on} \cdot P_{obst}. \end{aligned} \tag{7}$$

Neighboring FAPs can be used to determine the powers of undefined FAPs. By neighboring we mean their femtocell coverage is overlapped, thus the maximum distance between two neighboring FAPs is $2R_{femto}$. Thus the undefined FAP can be set to the power of its neighboring FAP without losing accuracy. Although these femtocells overlap and thus can sense the existence of each other, there's probably that no reliable link between these two FAPs to communicate. The neighboring procedure can be achieved by the assist of femtocell gateway.

The isolated probability for FAPs must be defined first and we can use Boolean model to resolve this issue.

Lemma 1: Consider homogeneous BM Ξ_{BM} in $\mathbb{R}^d$ with intensity λ. Let variable R denotes the radius of the circle coverage of the grains. Assume that $E[R^d] < \infty$. Then, the probability that a typical grain of radius R_0 is isolated is equal to

$$P^0\{B_0(R_0) \cap \Xi_{BM} = \emptyset\} = E[e^{-\lambda v_d \sum_{k=0}^{d} \binom{d}{k} R_0^{d-k} E[R^k]}], \tag{8}$$

and v_d is the volume of a unit-radius ball in $\mathbb{R}^d$.

In the paper, all the femtocell coverage radius is equal and two-dimensional space, that is, plane is considered. As a result, the isolated probability P_{iso} can be reduced to:

$$P_{iso} = e^{-4\pi \cdot R^2_{femto} \cdot \lambda_{femto}}. \tag{9}$$

Consequently, the probability of power-undefined FAPs in this case is:

$$\begin{aligned} P^1_{undef} &= P\{\text{FAP is isolated \& be obstructed \& no user}\} \\ &= P_{iso} \cdot P_{obst} \cdot P_{on}. \end{aligned} \tag{10}$$

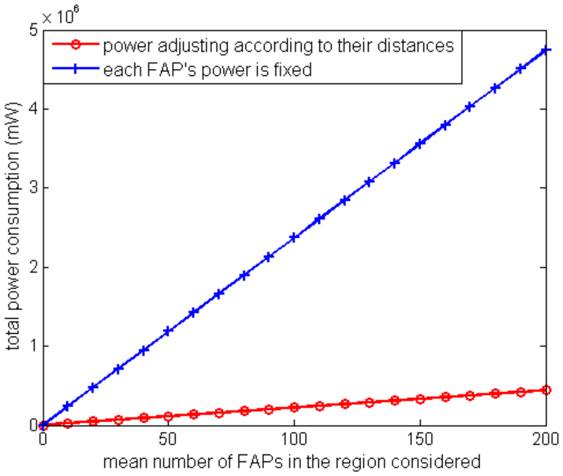

Figure 3. The total power consumed. Blue line represents the case when each FAP's power is fixed and set according to the worst situation while the red line represents that each FAP's power is set according to their distance from the MBS. In this figure, the MBS's transmit power P_m is $43dbm$, macro's radius R_{macro} $200m$, femtocell's radius R_{femto} is $20m$, forbid area's radius R_{forbid} is $50m$, outdoor path loss exponent α is 3, indoor path loss exponent β is 2.5, d_m is 10, d_f is 5, L_m, L_w, Lf and T all are equated to 1.

Consequently, if we use neighboring FAPs to assist defining the transmit power of a FAP that is obstructed from the MBS's signal, then the ratio of the reduced power-undefined probability is:

$$Ratio_{undef} = \frac{P_{undef} - P^1_{undef}}{P_{undef}} = 1 - P_{iso}. \tag{11}$$

IV. APPLICATION OF OUR MATHEMATICAL EXPRESSIONS

In this section, we will illustrate here how our mathematical expressions can be used to resolve power consumption.

Let us suppose the MBS's transmit power P_m is $43dBm$, macro's radius R_{macro} $200m$, femtocell's radius R_{femto} is $20m$, forbid area's radius R_{forbid} is $50m$, outdoor path loss exponent α is 3, indoor path loss exponent β is 2.5, d_m is 10, d_f is 5, L_m, L_w, Lf and T are equated to 1. It's obvious from figure 3 that the power reduction in the power adjusting case is enormous compared to the fixed case under our specific parameters but which are reasonable.

Next, we want to explore how the ratio of MBS's signal strength to FAP's when measured in the border (that is, T) affect the total power consumption. The ratio in the femtocell's border is the worst ratio among the entire femtocell coverage area. Letting T equals $0dB$, $3dB$, $5dB$ and $8dB$ and letting other parameters are the same as in the preceding. Figure 4 shows the change of total power consumption.

Figure 5 shows the corresponding ratio of the reduced power-undefined probability. In order to clarify the performance gain, we draw the mean number of femtocells in

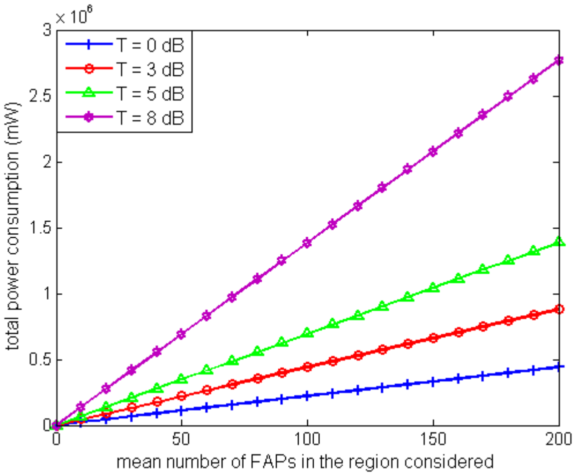

Figure 4. The total power consumption when T equals 0 dB, 3 dB, 5 dB and 8 dB, respectively. Other parameters are the same as in the preceding.

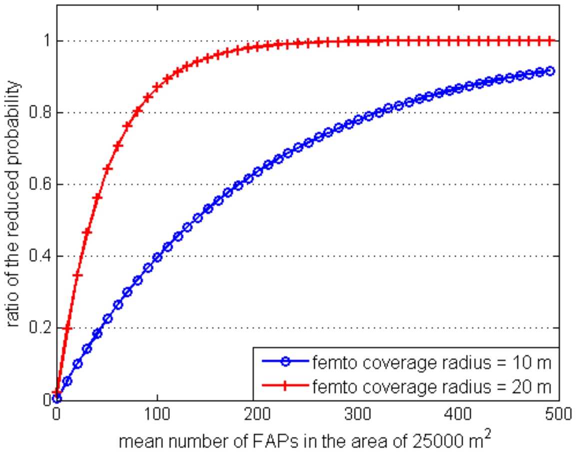

Figure 5. The ratio of the reduced power-undefined probability if we use neighboring FAPs to assist the power-undefined FAPs.

the area of $250000m^2$ on the abscissa rather the intensity λ_{femto}. It can be seen from figure 5 that with the femtocell density increasing, the power-undefined probability is decreasing which is in accordance with the intuition.

V. CONCLUSION

Firstly, we propose a power adjusting strategy for FAPs according to their relative distance from the macro base station and then quantitatively analyze its mean total power consumed in the femtocell network. In this stage, FAPs are regarded as terminals and evaluate their distance from the MBS's signal. There're possibilities that the signal strength from the MBS is obstructed such that the FAP can't achieve its distance. In order to combat this issue, neighboring FAPs are used to determine this power-undefined FAP. Thus the femtocell isolated probability should be pinned down first, which can be resolved by Boolean model (which is one branch of Stochastic Geometry). If there're no users in a specific FAP's coverage area, this FAP should turn down or off their transmit power and the probability that the FAP's "silent" probability is also achieved. Although we are merely focused on a typical signal propagation model, other signal propagation models can be used without losing any generality, thus provides a tractable framework about the power consumption in the femtocell network.

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