

# OPTIMAL TRANSMISSION DISTANCE OF MEAN PROGRESS AND MEAN TRANSPORT IN DEVICE-TO-DEVICE NETWORKS

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## Abstract

In this paper, stochastic geometry is used to quantitatively solve the issue of optimal transmission distance in device-to-device hybrid network. Two interference sources are taken into account: mobiles transmitting to base stations, and mobiles transmitting to other mobiles. Transmission distance is assumed equal in this paper, and we explore the optimal transmission distance of mean progress and mean transport, that is, the product of transmission distance and coverage probability, and the product of transmission distance and throughput, respectively. Parameters such as intensity of mobiles, the proportion of mobile regarded as device transmitters, signal-to-interference-plus-noise ratio requirement, ratio of transmit power of macro terminals to mobile devices, and path loss exponent, are considered in our mathematical expressions. In this paper, we illustrate how the stochastic geometry can be used in studying the device-to-device issues, and how the optimal transmission distance changes regarding the parameters.

## 1 Introduction

Device-to-device (D2D) communication extends the prevailing communication methods of mobile devices communicating by directing data exchange between devices. This type of communication is motivated by new or existing services, which benefit from direct connectivity between devices. If information is only of local interest, it may be beneficial to connect devices directly instead of through the operators' networks. Security of networks may also benefit from improved reliability, robustness, and coverage provided by device-to-device communications, illustrating another use case where direct communication between devices can play a key role [1].

Several device-to-device communication technologies already exist, including Bluetooth and Wi-Fi. These technologies typically operate in the unlicensed ISM bands and are hence subject to uncontrolled interference, which can reduce user experience. To guarantee device-to-device communications with quality-of-service, it can be beneficial to operate in licensed spectrum. Further, the network can provide benefits to the device-to-device communication by assisting it in various ways, e.g., with device discovery, radio-resource assignment, synchronization, and security, regardless of operating in licensed or unlicensed bands [1].

One key challenge in D2D networks is mode selection. In a cellular network system one way to increase its capacity is to allow direct communication between closely located user devices when they are communicating with each other instead of conveying data from one device to the other via operators' core network. The problem is then when shall the network assign direct communication mode and when not [2-4].

Authors in [5] and [6] explore the design aspects of network assisted device-to-device communications which is compatible with the 3GPP Long Term Evolution system. Authors in [7] consider rate splitting and interference cancelation in D2D communications underlying a cellular network, in which message is split into a private and a public part, as in Han-Kobayashi scheme.

Femtocell system is also regarded as one of the key technologies in the 4G LTE-A. Both D2D and femtocell systems can offload the traffic from macrocell systems, improve the spectrum efficiency, and benefit both mobile users and operators. Hence, the authors in [8] explore the resource management issues in the heterogeneous networks comprised of D2D, femtocell and macrocell, whereas in [9], resource allocation based on price auction is proposed. In [10], a distributed resource allocation scheme is proposed and analyzed.

Although there are a lot of papers investigating performance analysis of D2D networks, those papers tend to ignore the randomness of the locations of mobile devices, which brings us a lot of difficulties to analyze and compare. In order to

quantitatively analyze the performance, stochastic geometry (especially Poisson point process) could be used. One advantage of this approach is the ability to capture the non-uniform layout. Additionally, tractable expressions can be drawn from the Poisson model, leading to more general performance characterizations and intuition [11].

In this paper, we are mainly focused on the optimal transmission distance of mean progress and mean transport, that is, the product of transmission distance and coverage probability, and the product of transmission distance and throughput, respectively. The contribution of our paper is that we apply stochastic geometry to model the locations of mobile devices, and combat one typical issue of D2D networks, that is, the optimal transmission distance of mean progress and mean transport. Although the system model is simplified, the changing tendency of optimal transmission distance is intuitive and tractable. Other issues of D2D networks can also be solved by similar reasoning with the aid of stochastic geometry.

This paper is organised as follows: first, homogenous Poisson point process is introduced, and how it can be applied to D2D networks is clarified; then, our system model is given and relative assumptions are explained; after mathematical expressions have been achieved, simulation result is given to illustrate the change of optimal transmission distance.

## 2 System model

In this section, our system model is given and relevant parameters are clarified. First, the Poisson point process is introduced which is applied in this paper to model the locations of mobile devices. The definition of Poisson point process is given below.

Definition 1: Let  $\Lambda$  be a locally finite non-null measure on  $\mathbb{R}^d$ . The Poisson point process  $\Phi$  of intensity measure  $\Lambda$  is defined by means of its finite-dimensional distributions:

$$P\{\Phi(A_1) = n_1, \dots, \Phi(A_k) = n_k\} = \prod_{i=1}^k (e^{-\Lambda(A_i)} \frac{\Lambda(A_i)^{n_i}}{n_i!}), \quad (1)$$

for every  $k = 1, 2, \dots$  and all bounded, mutually disjoint sets  $A_i$  for  $i = 1, \dots, k$ . If  $\Lambda(dx) = \lambda dx$  is a multiple of Lebesgue measure (volume) in  $\mathbb{R}^d$ , then  $\Phi$  is a homogeneous Poisson point process and  $\lambda$  is the intensity.

The realisation of Poisson point process (p.p.) can be constructed as follows. If the region considered has an area of  $|A|$ , and then the mean number of points in this region is  $\lambda \cdot |A|$ . The homogeneous Poisson p.p. can be simply produced by a random variable  $X$  following Poisson distribution with parameter  $\lambda \cdot |A|$ , and each point is distributed uniformly among the region.

In this article, we assume that the locations of interference mobiles follow a homogeneous Poisson point process with parameter  $\lambda$ . Mobiles are further divided into two categories: ones that communicate with base station which we will refer to macro terminals, and ones that communicate with other mobile devices which we will refer to mobile devices in the rest of this article. The transmit power of macro terminals is  $p_m$ , while the transmit power of mobile devices is  $p_d$ . The ratio of mobile devices is  $p$ . According to independent thinning theory of Poisson point process, the locations of macro terminals and mobile devices follow homogenous Poisson point process with parameter  $(1-p) \cdot \lambda$  and  $p \cdot \lambda$ , respectively. The channel follows Rayleigh fading and the noise follows white noise with constant power density  $w$ .

The target mobile device (at the origin) receives signals from one transmit mobile device located at a distance of  $r$ , all other macro terminals and mobile devices act as interferers. The difference between macro terminals and mobile devices lies in their transmit power. Fig. 1 is a snapshot of interference mobiles following homogeneous Poisson point process. In Fig. 1, the black dot represents the target mobile device at the origin; blue dots represent the macro terminals, while the red dots represent the mobile devices. In this figure,  $\lambda = 8 \times 10^{-4}$ ,  $p = 0.4$ .

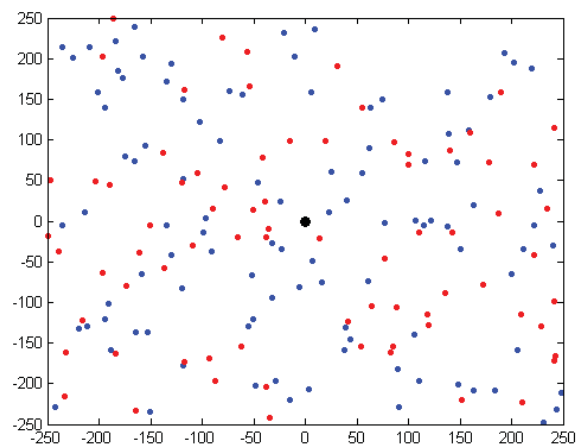


Figure 1. One snapshot of interference mobiles following homogeneous Poisson point process.

In this paper, we assume that each receiving mobile device is receiving signals from transmitting mobile devices with a distance of  $r$ . We will explore how the optimal distance  $r$  changes with the system parameters, that is, density of interference mobiles  $\lambda$ , ratio of mobile devices  $p$ , ratio of transmit power of macro terminals to mobile devices  $p_m / p_d$ , SINR requirement  $T$ , and path loss exponent  $\alpha$ .

### 3 Mean progress and mean transport in a typical transmission

In this section, we will study the performance of mean progress and mean transport in a typical transmission with the optimal distance  $r$ . The mean progress of a typical transmission is defined as  $r \cdot P_c(r, \lambda, p, T, p_m, p_d, \alpha)$ , where  $P_c(r, \lambda, p, T, p_m, p_d, \alpha)$  is the coverage probability of the typical transmission, and the mean transport of a typical transmission is defined as  $r \cdot \tau(r, \lambda, p, T, p_m, p_d, \alpha)$ , in which  $\tau(r, \lambda, p, T, p_m, p_d, \alpha)$  is the throughput of the typical transmission.

#### 3.1 Mean progress in a typical transmission

In order to calculate the mean progress, the coverage probability of a typical transmission should be first derived. The coverage probability is defined as

$$P_c(r, \lambda, p, T, p_m, p_d, \alpha) @P[SINR > T]. \quad (2)$$

Regarding random channel effects such as fading and shadowing, we assume that all mobiles experience Rayleigh fading with mean 1, and employ a constant transmit power of  $p_m$  and  $p_d$  for macro terminals and mobile devices, respectively. Thus, as to the target mobile device, the received interference power from a macro terminal with a distance  $d$  is  $h_m d^{-\alpha}$  where the random variable  $h_m$  follows an exponential distribution with mean  $p_m$ , while the received interference power from a mobile device with a distance  $d$  is  $h_d d^{-\alpha}$  where the random variable  $h_d$  follows an exponential distribution with mean  $p_d$ .

The signal received at the target mobile device is

$$h_d r^{-\alpha}, \quad (3)$$

and the interference received is

$$I_d + I_m + w, \quad (4)$$

where  $I_d = \sum_{i \in \Phi_d} h_{di} R_i^{-\alpha}$  and  $I_m = \sum_{i \in \Phi_m} h_{mi} R_i^{-\alpha}$  representing the interference from the layer of mobile devices  $\Phi_d$  and macro terminals  $\Phi_m$ , respectively, and  $R_i$  is the distance from  $i^{th}$  mobile to the origin. Hence, the SINR experienced by the target device is

$$SINR = \frac{h_d r^{-\alpha}}{I_d + I_m + w}, \quad (5)$$

and

$$\begin{aligned} P\{SINR > T\} &= P[h_d > T \cdot (I_d + I_m + w) \cdot r^\alpha] \\ &= \exp\{-1/p_d \cdot T \cdot r^\alpha \cdot (I_d + I_m + w)\} \\ &= \exp\{-1/p_d \cdot T \cdot r^\alpha \cdot w\} \\ &\quad \cdot L_{I_d}(1/p_d \cdot T \cdot r^\alpha) \cdot L_{I_m}(1/p_m \cdot T \cdot r^\alpha) \end{aligned} \quad (6)$$

where  $L_{I_d}(s)$  and  $L_{I_m}(s)$  is the Laplace transform of random variable  $I_d$  and  $I_m$  evaluated at  $s$  conditioned on the distance to the origin, respectively. Next, we will evaluate  $L_{I_d}(s)$  and  $L_{I_m}(s)$ .

$$\begin{aligned} L_{I_d}(s) &= E_{I_d}[e^{-s I_d}] = E[\exp(-s \cdot \sum_{i \in \Phi_d} h_{di} R_i^{-\alpha})] \\ &= \exp(-2\pi\lambda p \cdot \int_0^\infty [1 - E_{hd}(e^{-s \cdot h_d \cdot v^{-\alpha}})] v dv) \end{aligned} \quad (7)$$

where an identity has been used that for Poisson point process,

$$L_\Phi(f) = \exp(-\int_0^\infty (1 - e^{-f(x)}) \Lambda(dx)).$$

Because  $h_d \sim \exp(1/p_d)$ , then

$$\begin{aligned} E_{hd}(e^{-s \cdot h_d \cdot v^{-\alpha}}) &= \int_0^\infty e^{-s \cdot h_d \cdot v^{-\alpha}} \cdot \frac{1}{p_d} \cdot e^{-1/p_d} dh_d \\ &= \frac{1/p_d v^{-\alpha}}{s + 1/p_d v^{-\alpha}} \end{aligned} \quad (8)$$

Substituting Equation (8) into Equation (7) comes

$$L_{I_d}(s) = \exp\{-\lambda p (s \cdot p_d)^{2/\alpha} \cdot K(\alpha)\}, \quad (9)$$

where

$$K(\alpha) = \frac{2\pi^2}{\alpha \sin(2\pi/\alpha)}. \quad (10)$$

Similarly,

$$L_{I_m}(s) = \exp\{-\lambda(1-p)(s \cdot p_m)^{2/\alpha} \cdot K(\alpha)\}. \quad (11)$$

Combining Equation (9) and Equation (11) into Equation (6) we have

$$\begin{aligned} P_c(r, \lambda, p, T, p_m, p_d, \alpha) &= P[SINR > T] \\ &= \exp\{-1/p_d \cdot T \cdot r^\alpha \cdot w\} \\ &\quad \cdot \exp\{-\lambda \cdot T^{2/\alpha} \cdot r^2 \cdot G(\alpha, p_m, p_d)\} \end{aligned} \quad (12)$$

where

$$G(\alpha, p_m, p_d) = [(1-p) \cdot (\frac{p_m}{p_d})^{2/\alpha} + p] \cdot K(\alpha). \quad (13)$$

It is obvious that the coverage probability  $P_c(r, \lambda, p, T, p_m, p_d, \alpha)$  is maximized at  $r = 0$  which is not practical in most cases. In this paper, we consider the mean progress made in a typical transmission as in [11], that is

$$prog(r, \lambda, p, T, p_m, p_d, \alpha) = r \cdot P_c(r, \lambda, p, T, p_m, p_d, \alpha). \quad (14)$$

### 3.1 Mean transport in a typical transmission

As in the case of mean progress, in order to calculate the mean transport, the throughput of a typical transmission should be first derived. The throughput of the channel from transmitter  $X_i$  to its receiver  $y_i$  is defined as

$$C_i = \log(1 + SINR_i). \quad (15)$$

The throughput of a typical transmitter is

$$\begin{aligned} \tau(r, \lambda, p, T, p_m, p_d, \alpha) &= E[\log(1 + SINR_0)] \\ &= \int_0^\infty P\{\log(1 + SINR_0) > t\} dt \\ &= \int_0^\infty P\{SINR > e^t - 1\} dt \\ &= \int_0^\infty P_c(r, \lambda, p, e^t - 1, p_m, p_d, \alpha) dt \\ &= \int_0^\infty \frac{P_c(r, \lambda, p, v, p_m, p_d, \alpha)}{v + 1} dv \end{aligned} \quad (16)$$

Substituting Equation (12) into Equation (16) comes

$$\begin{aligned} \tau(r, \lambda, p, T, p_m, p_d, \alpha) \\ = \frac{\alpha}{2} \int_0^\infty e^{-\lambda G(\alpha) r^{2v}} \cdot e^{-1/p_d \cdot r^\alpha \cdot v^{\frac{\alpha}{2}}} \cdot \frac{v^{\frac{\beta}{2}-1}}{1 + v^2} dv. \end{aligned} \quad (17)$$

It is obvious that the throughput  $\tau(r, \lambda, p, T, p_m, p_d, \alpha)$  is maximized at  $r = 0$  which is not practical in most cases. In this paper, we consider the mean transport of a typical transmission as in [11], that is

$$trans(r, \lambda, p, T, p_m, p_d, \alpha) = r \cdot \tau(r, \lambda, p, T, p_m, p_d, \alpha). \quad (18)$$

## 4 Optimal transmission distance

We have obtained the equations for mean progress and mean transport of a typical transmission. In this section, the optimal transmission distance  $r$  will be discussed and see how its value changes with other parameters.

### 4.1 Optimal transmission distance of mean progress

We denote by

$$r_{\max}^{prog} = \arg \max_{r \geq 0} prog(r, \lambda, p, T, p_m, p_d, \alpha) \quad (19)$$

the best transmission distance whenever such a value exists.

The optimal value of  $r_{\max}^{prog}$  can be achieved simply by differentiation of the function of Equation (14) and let it equals to zero, and the result is

$$r_{\max}^{prog} = \left( \frac{1}{2\lambda \cdot T^{2/\alpha} \cdot G(\alpha, p_m, p_d)} \right)^{1/2}, \quad (20)$$

where  $G(\alpha, p_m, p_d)$  is given in Equation (13).

### 4.2 Optimal transmission distance of mean transport

Same as the case of mean progress, we denote by

$$r_{\max}^{trans} = \arg \max_{r \geq 0} trans(r, \lambda, p, T, p_m, p_d, \alpha) \quad (21)$$

the best transmission distance whenever such a value exists.

The optimal value of  $r_{\max}^{trans}$  can be achieved simply by differentiation of the function of Equation (18) and let it equal to zero, and the result is

$$r_{\max}^{trans} = \left( \frac{y(\alpha)^*}{\lambda \cdot G(\alpha, p_m, p_d)} \right)^{1/2}, \quad (22)$$

where  $y(\alpha)^*$  is the unique solution of the integral equation

$$\int_0^\infty e^{-yv} \frac{v^{\alpha/2-1}}{1 + v^{\alpha/2}} dv = 2y \int_0^\infty e^{-yv} \frac{v^{\alpha/2}}{1 + v^{\alpha/2}} dv. \quad (23)$$

## 5 Simulation results

In this section, elaborative simulation results will be given. Specifically, we will explore how the transmission distance changes with the intensity of mobile  $\lambda$ , the proportion of mobile devices  $p$ , SINR requirement  $T$ , ratio of transmit power of macro terminals to mobile devices  $p_m / p_d$ , and path loss exponent  $\alpha$ .

Fig. 2, Fig. 3, and Fig. 4 show the optimal transmission distance for the mean progress and for the mean transport when path loss exponent  $\alpha$  is 3, 4, and 5, respectively. Comparing among those three figures, we can understand how the optimal distance changes. In these figures, red lines represent the cases of mean transport, while the blue lines represent the cases of mean progress. The solid, dashed, and dotted lines present the case when the proportion of mobile devices is 0.9, 0.5, and 0.1, respectively. The ratio of transmit power of macro terminals to mobile devices is 10. The intensity of mobiles is changing from 0.0004 to 0.002, which is equivalent to 100 and 500 mobiles when the considered area is  $500 \times 500$  square meters. It can be seen from these figures that the optimal transmission distance is more stable in the mean progress cases than in the mean transport cases. Specially, about 30 meters is optimal for mean progress no matter what other parameters change. In addition, larger transmission distance is preferred in the mean transport cases because we take Shannon channel into account (Section 3.2).

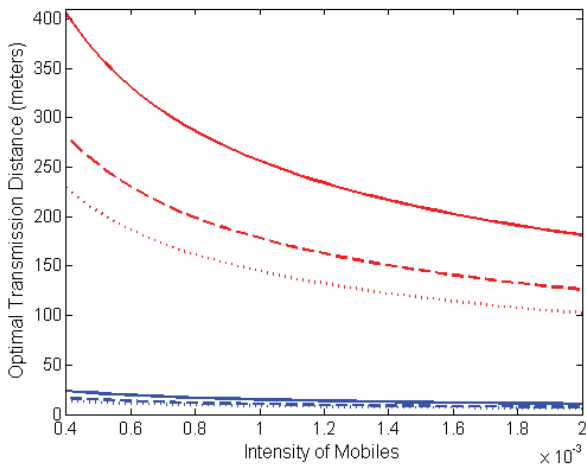


Figure 2. Optimal transmission distance for mean progress and mean transport when path loss exponent  $\alpha = 3$ .

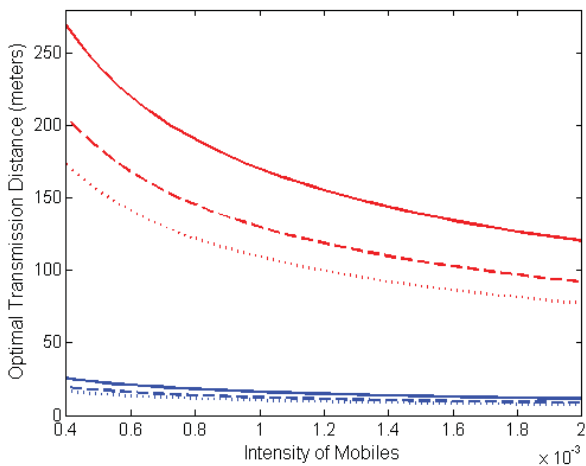


Figure 3. Optimal transmission distance for mean progress and mean transport when path loss exponent  $\alpha = 4$ .

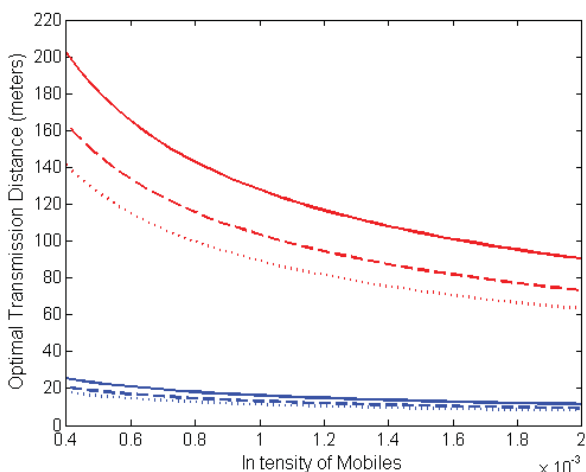


Figure 4. Optimal transmission distance for mean progress and mean transport when path loss exponent  $\alpha = 5$ .

## 6 Conclusion

In this paper, stochastic geometry is used to quantitatively solve the issue of optimal transmission distance in device-to-device hybrid network.

First, homogeneous Poisson point process is introduced, and how it is used in combating issues of device-to-device communication is illustrated. After explaining our system model comes the main part of this paper which derives the mathematical expressions for optimal transmission of mean progress and mean transport. It has been shown that the optimal transmission distance for mean progress is more stable than that of mean transport. Specially, about 30 meters is optimal for mean progress no matter what other parameters change.

This paper illustrates how stochastic geometry can be applied to effectively solve issues of device-to-device communications. Relative issues in D2D networks can be solved in similar lines of reasoning.

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